20.2 Plotting First-Motion Fault Plane Solutions

The problem now is to plot in some type of projection the pattern of the first motions of P or S waves whose rays emanate from the shear dislocation source and are refracted through the earth according to Snell's law and the velocity model:
Recall:

\[ \frac{\sin i}{V} = p \quad \text{flat layers} \]

or

\[ \frac{r \sin i}{V} = p \quad \text{for a spherical earth of radius } r \text{ to a particular layer} \]

To facilitate this process let us enclose the fault in an imaginary sphere whose radius is very small compared to the epicentral distances, called the focal sphere. This allows us to consider the rays in the focal sphere as straight lines.
\( S_i \) are the intersection of the ray, that eventually propagates to station \( S_i \) on the earth's surface, with the focal sphere. Thus, we can transform points recorded on the surface of the earth backward to points on the focal sphere. Then we can plot their projection, \( B_a \), onto the plane \( x-y \) using the top of the focal sphere as the pole of a stereographic projection.

For a unit radius circle:
\[
OB_a = \tan \left( \frac{i}{2} \right)
\]

where \( i \) = angle of incidence at the focus. Hence, we can map all stations on the earth's surface back onto the plane \( x-y \) corresponding to their focal sphere positions. In this projection, azimuths on the earth's surface map into equal azimuths on the stereo plot and epicentral distances, \( \Delta_i \), map into angles of incidence \( i_i \). The more distant stations will have smaller \( i \)'s and closer stations larger \( i \)'s. Note that we could also use the other poles of the projection, i.e. the bottom pole if we desired.

For upward propagating rays, \( i > 90^\circ \) thus we use an upper-hemisphere (lower pole) plot, For \( i < 90^\circ \) use a lower hemisphere plot.
Hence rays propagating upward must be projected in the upper hemisphere. Rays initially propagating downward, but then refract upward, are plotted in the lower hemisphere. Generally then for small \( \Delta \)'s, i.e. for microearthquake surveys the rays plot into the upper hemisphere; for teleseismic earthquakes rays are plotted into the lower-hemisphere.