17.4 Array Analysis for Hypocenters Outside of a Network

As we have seen, hypocenters located outside of an array have very poor locations (refer to homework). Hence, we must be able to locate events outside the array, at least within one to two array-radii. We can do this by ray-tracing back to a hypocenter. We need a direction (azimuth), an angle of emergence, $i_o$, and a distance.

Consider three stations, $S_i, i=1, 2, 3$, with a plane wave progressing across the array with an azimuth $\psi$. The wave sweeps across the array with an apparent velocity $V$.

![Diagram of ray-tracing]

\[ \sin i_o = \frac{V_o |t_2 - t_1|}{x} \]

\[ = \frac{V_o}{V_{app}} \]

so the apparent velocity, $V_{app}$ is:

\[ V_a = \frac{V_o}{\sin i_o} \]

But now consider the array below:
We want to obtain $\psi$, azimuth, and $V$, the apparent velocity, across the array.

Using:

\[ t_2 - t_1 = l_2 \sin(\theta_2 - \phi) / V_a \]
\[ t_3 - t_1 = l_3 \sin(\theta_3 - \phi) / V_a \]

17.4.1

and eliminating $V_a$ by setting

\[ \gamma = \frac{t_3 - t_1}{t_2 - t_1} \cdot \frac{l_2}{l_3} \]

then dividing equations 17.4.1 gives:

\[ \sin(\theta_3 - \phi) = \gamma \sin(\theta_2 - \phi) \]
Then, expanding and solving for $\sin (\theta_3 - \phi)$

$$\tan \phi = \frac{\sin \theta_3 - \gamma \sin \theta_2}{\cos \theta_2 - \gamma \cos \theta_2}$$

Thus,

$$\phi = \tan^{-1} \left( \frac{\sin \theta_3 - \gamma \sin \theta_2}{\cos \theta_3 - \gamma \cos \theta_2} \right)$$

$$V = \frac{l_2 \sin (\theta_3 - \theta)}{t_0 - t_1}$$

Now for a homogeneous body we can obtain the origin time by

$$V_s = \frac{X}{t_s}, \quad V_p = \frac{X}{t_p}$$

then

$$t_0 = t_{pi} - t_{Tp} \quad \text{t}_{Tp} = \text{travel time of P}$$

$$t_{pi} = \text{arrival time of P}$$

$$t_p = t_{tp}$$

and

$$(t_s - t_p) = X \left( \frac{1}{V_s} - \frac{1}{V_p} \right)$$

$$t_p = \frac{X}{V_p} = \frac{t_s - t_p}{V_{s-p} V_p}$$

$$= X \left( \frac{V_p - V_s}{V_p V_s} \right)$$

$$\downarrow$$

$$V_{s-p}$$

for a Poisson's ratio $= 0.235$, $V_p = 2.9 V_s$, and

$$V_{s-p} = \frac{V_p V_s}{V_p - V_s} = 1.43 V_p = \text{for } V_p = 6.0 \quad V_{sp} = 8.6$$

so 17.4.2 becomes

$$t_0 = t_{pi} - 1.43 (t_s - t_p)$$

17.4.3
Now for a one-layer case.

The total travel time, $t_k$, can also be expressed in the following form by manipulating 17.4.3:

$$t_k = 1.43 (t_s - t_p)$$

hence,

$$x = (V)(1.43)(t_s - t_p) = Vt_k$$

and

$$
\cos i_o = \frac{h_2}{x}
$$

so the depth, $h_2$, can be expressed as:

$$h_2 = V (1.43) (t_s - t_p) \cos i_o$$

We may further develop this technique for several layers, but it becomes rather complicated rather quickly.